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ESTIMATING THE FILTRATION CHARACTERISTICS OF RESERVOIRS FROM DATA OF NONSTATIONARY STUDIES OF HORIZONTAL WELLS

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Regularization methods are used to construct computational algorithms for the interpretation of results from hydrodynamic studies of horizontal wells that provide estimates of the reservoir anisotropy, reservoir pressure, and the dependence of the in-place permeability on pressure. In contrast to graphic analytic methods, the proposed approach does not require the identification of flow regimes.

Key words: *inverse problem, regularization, horizontal well, reservoir pressure, anisotropy, permeability.*

The problems of determining the filtration characteristics of an oil-and-gas reservoir from data of nonstationary hydrodynamic studies belong to the class of inverse problems of underground hydromechanics. A feature of these problems is that additional information depends on the capabilities of field experiments.

Existing graphical analytic methods for interpreting pressure build-up curves obtained in experiments on horizontal wells are based on the replacement of the fluid inflow to a horizontal wellbore by a sequence of plane flows [1].

In the present paper, the fluid inflow to a horizontal wellbore is simulated numerically, i.e., the three-dimensional problem of fluid filtration to a horizontal well is solved. The results of hydrodynamic studies of horizontal wells are interpreted using a computational algorithms constructed on the basis of regularization methods. Calculations are performed of the in-place permeability as a function of pressure (a nonlinearly elastic filtration mode), the permeability of an anisotropic reservoir, and the reservoir pressure (an elastic filtration mode). Calculations results are given.

1. Filtration processes in oil-and-gas reservoirs with pressure dependent permeability have been the subject of active research. This is motivated by studies of the effect of well operation modes on the filtration characteristics of reservoirs [2].

The inverse problem consists of determining the parameter $s(p) = k(p)/\mu$ for the filtration process described by the equation

$$\beta^* \frac{\partial p}{\partial t} = \nabla(s(p)\nabla p), \qquad 0 < t \leqslant T, \quad (x, y, z) \in V \tag{1.1}$$

subject to the initial condition

$$p(x, y, z, 0) = p_0(x, y, z) \tag{1.2}$$

and the boundary conditions

$$(s(p)\nabla p, \boldsymbol{n})\Big|_{\partial V_1} = 0; \tag{1.3}$$

$$p\Big|_{\partial V_2} = p_{\rm res}; \tag{1.4}$$

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$$(s(p)\nabla p, \boldsymbol{n})\Big|_S = q(x,y,z,t), \tag{1.5}$$

where s is the oil mobility, k is the reservoir permeability, μ is the fluid dynamic viscosity, β^* is the elastic capacity factor of the reservoir, p is the pressure, p_{res} is the reservoir pressure, $q(x,y,z,t)$ is the fluid influx per unit surface of the horizontal wellbore, V is the filtration region bounded by the outer surface $\partial V = \partial V_1 \cup \partial V_2$, S is the surface of the horizontal wellbore, and $\boldsymbol{n}$ is the unit normal vector.

Experimental relations between the permeability and pressure can be approximated by monotonic and convex functions [2]. The solution of the inverse problem taking into account the constraints on the desired function (monotonicity and convexity) is sought using the functional minimum condition

$$\inf_{s\in D} J(s), \qquad J(s) = \int\limits_0^T (p_{\text{ob}}(t) - p_{\text{calc}}(t))^2\,dt, \tag{1.6}$$

where $p_{\text{ob}}(t)$ and $p_{\text{calc}}(t)$ are the observed and calculated well pressures, T is the time of the experiment, and D is the set of admissible functions that satisfy the conditions

$$0 < s_{\min} \leqslant s(p) \leqslant s_{\max}, \qquad s_p(p) \geqslant 0, \qquad s_{pp}(p) \geqslant 0, \qquad p \in [M_1, M_2], \tag{1.7}$$

$$M_1, M_2, s_{\min}, s_{\max} = \text{const} > 0.$$

Using the small-perturbation method and the condition of stationarity of the Lagrange functional, we obtain the following expression for the functional gradient:

$$(\nabla J, \delta s) = -\int\limits_0^T \int\limits_V (\nabla\psi, \nabla p)\delta s\,dV\,dt.$$

Here $\psi(x,y,z,t)$ is a solution of the conjugate problem

$$\begin{gathered} -\beta^* \frac{\partial \psi}{\partial t} = \nabla(s(p)\nabla\psi) - (\nabla p, s_p \nabla\psi), \qquad 0 \leqslant t < T, \qquad (x,y,z) \in V, \\ \psi(x,y,z,T) = 0, \qquad (s(p)\nabla\psi, \boldsymbol{n})\Big|_{\partial V_1} = 0, \qquad \psi\Big|_{\partial V_2} = 0; \end{gathered} \tag{1.8}$$

$$(s(p)\nabla\psi, \boldsymbol{n})\Big|_S = q^*(x,y,z,t). \tag{1.9}$$

Problems (1.1)–(1.5) and (1.8), (1.9) are solved numerically using the finite-difference method. The quantity $q^*(x,y,z,t)$ is determined by solving the problem (1.1)–(1.5). To approximate the coefficient $s(p)$, we consider the set of grid functions $\tilde{s} = (s_0, \ldots, s_l, \ldots, s_{N_l})$ defined at the nodes of the grid $\bar{\omega}_\sigma = \{p_l;\ M_1 = p_0 < p_1 < \ldots < p_{N_l} = M_2,\ p_l - p_{l-1} = \sigma_l\}$, such that for $p^n_{ijk} \in [p_{l-1}, p_l)$

$$s(p^n_{ijk}) = s_{l-1} + \frac{p^n_{ijk} - p_{l-1}}{\sigma_l}(s_l - s_{l-1}), \qquad l = 1, \ldots, N_l,$$

where $s_l = s(p_l)$; p^n_{ijk} is the pressure in the mesh (i,j,k) of a finite-difference grid at the nth time interval.

The discrete analog of the variational problem (1.6) is the following problem of nonlinear programming:

$$\min_{\tilde{s}\in\tilde{D}} J(\tilde{s}), \qquad J(\tilde{s}) = \sum_{n=1}^{N_\tau} \tau_n (p^n_{\text{ob}} - p^n_{\text{calc}})^2. \tag{1.10}$$

Here τ_n is the step of the time grid and $\tilde{D}$ is the set of grid functions $\tilde{s} = (s_0, \ldots, s_l, \ldots, s_{N_l})$ that satisfy the constraints

$$\begin{gathered} 0 < s_{\min} \leqslant s_l \leqslant s_{\max}, \quad l = 0, \ldots, N_l, \qquad s_{l+1} \geqslant s_l, \quad l = 0, \ldots, N_l - 1, \\ \frac{s_{l+1} - s_l}{\sigma_{l+1}} \geqslant \frac{s_l - s_{l-1}}{\sigma_l}, \quad l = 1, \ldots, N_l - 1. \end{gathered} \tag{1.11}$$

Conditions (1.11) are discrete analogs of conditions (1.7).

The functional (1.10) is minimized subject to constraints (1.11) using the conjugate gradient projection method [3, 4]. The calculation of the gradient of the functional in each step of the iterative process includes the solution of the boundary-value problems (1.1)–(1.5), (1.8), and (1.9).

2. The majority of oil-and-gas reservoirs have layered structure due to the features of the sedimentation process. In such reservoirs, the filtration characteristics in the plane of the layers differ from the properties in a direction perpendicular to the layers. To estimate the reservoir anisotropy, we consider the following inverse problem: it is required to determine the principal values of the tensor of the permeability coefficients k_x, k_y, and k_z and the reservoir pressure $p_{\rm res}$ for the nonstationary filtration process described by the differential equation

$$\mu\beta^* \frac{\partial p}{\partial t} = k_x \frac{\partial^2 p}{\partial x^2} + k_y \frac{\partial^2 p}{\partial y^2} + k_z \frac{\partial^2 p}{\partial z^2}, \qquad 0 < t \leqslant T, \quad (x, y, z) \in V$$

with the initial condition

$$p(x, y, z, 0) = p_0(x, y, z)$$

and the boundary conditions

$$(\boldsymbol{w}, \boldsymbol{n})\Big|_{\partial V_1} = 0, \qquad p\Big|_{\partial V_2} = p_{\rm res}, \qquad -(\boldsymbol{w}, \boldsymbol{n})\Big|_{S} = q(x, y, z, t),$$

where $\boldsymbol{w}$ is the filtration velocity.

The solution of the inverse problem reduces to the minimization of the functional

$$J(k_x, k_y, k_z, p_{\rm res}) = \int_0^T (p_{\rm ob}(t) - p_{\rm calc}(t))^2 \, dt. \tag{2.1}$$

The iterative sequence for the minimization of the functional (2.1) is constructed using the gradient descent method [5, 6]. The gradients of the functional for the corresponding components of the permeability tensor and reservoir pressure are calculated by the formulas

$$J'_{k_x} = -\int_0^T \int_V \frac{\partial \psi}{\partial x} \frac{\partial p}{\partial x} \, dV \, dt, \quad J'_{k_y} = -\int_0^T \int_V \frac{\partial \psi}{\partial y} \frac{\partial p}{\partial y} \, dV \, dt, \quad J'_{k_z} = -\int_0^T \int_V \frac{\partial \psi}{\partial z} \frac{\partial p}{\partial z} \, dV \, dt,$$

$$J'_{p_{\rm res}} = -\int_0^T \int_{\partial V_2} \Big(\frac{k_x}{\mu} \frac{\partial \psi}{\partial x} n_x + \frac{k_y}{\mu} \frac{\partial \psi}{\partial y} n_y + \frac{k_z}{\mu} \frac{\partial \psi}{\partial z} n_z \Big) \, d\sigma \, dt,$$

where the function $\psi(x, y, z, t)$ is a solution of the corresponding conjugate problem; n_x, n_y, and n_z are the direction cosines of the normal to the surface ∂V_2.

3. In the calculations, the filtration region V was specified as a parallelepiped. Then, ∂V_1 is the roof and base surface of the reservoir and ∂V_2 are the lateral faces of the reservoir. The origin of the Cartesian coordinate system is at the beginning of the axis of the horizontal well bore, and the x axis is directed along the axis of the horizontal well. The influx $q(x, y, z, t)$ in (1.5) is calculated under the assumption that the pressure on the surface of the horizontal wellbore is constant. In this case, the production rate of the horizontal well is given by

$$Q = \int_S q(x, y, z, t) \, ds.$$

The quantity $q^*(x, y, z, t)$ in (1.9) is calculated under the assumption that function $\psi(x, y, z, t)$ is constant on the surface S. In this case,

$$\int_S q^*(x, y, z, t) \, ds = 2(p_{\rm ob}(t) - p_{\rm calc}(t)).$$

The right side of this expression is obtained using the condition of stationarity of the Lagrange functional.

In the calculations, the parabolic equation was approximated by an implicit finite-difference scheme. The calculation parameters were chosen such that the results were not affected by the use of a finer grid [7].

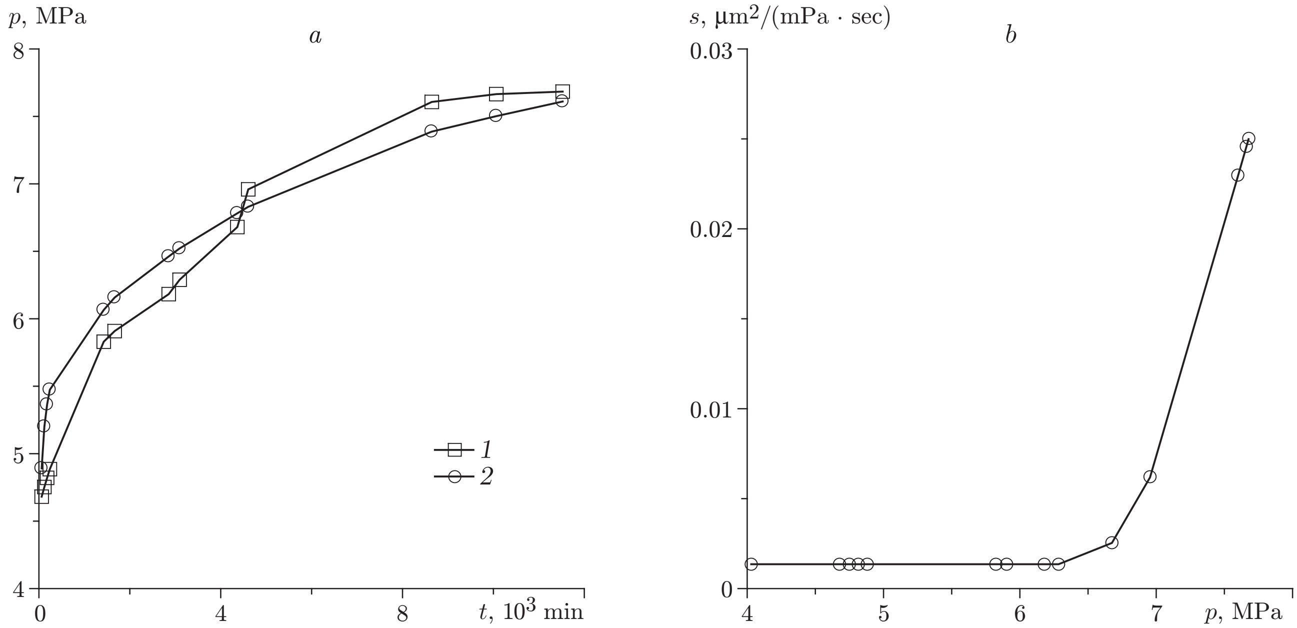

Fig. 1. Pressure build-up curve $p(t)$ (a) and calculated relation $s(p)$ (b) for a horizontal well No. 13473: 1) experiment; 2) calculation.

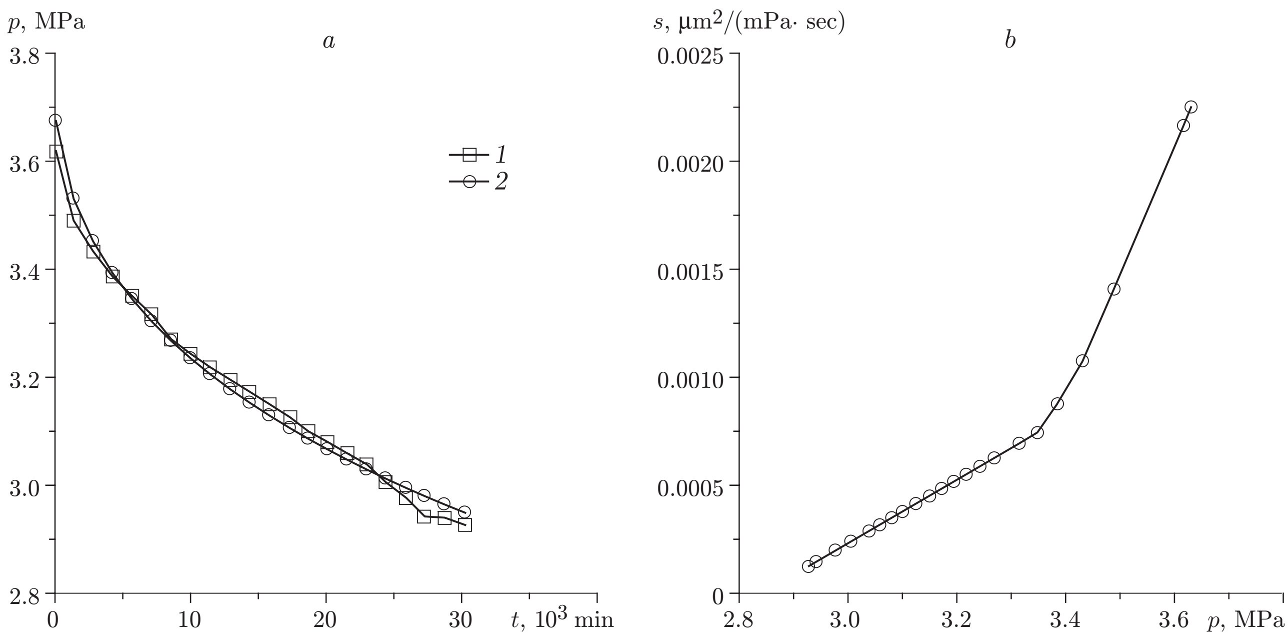

Fig. 2. Scavenging curve $p(t)$ (a) and calculated curve of $s(p)$ (b) for horizontal well No. 1947: 1) experiment; 2) calculation.

Figure 1a shows the pressure build-up curve for horizontal well No. 13473 of the Shegurcha field. The well production rate before shutdown is $Q = 5.9 \cdot 10^{-5}$ m^3/sec, the length of the horizontal portion of the well is 204 m, the oil viscosity is $\mu = 25$ mPa $\cdot$ sec, and the reservoir thickness is 22 m. As the initial approximation of the reservoir pressure, we used the last point on the pressure build-up curve ($p^0_{\text{res}} = 7.685$ MPa). For an anisotropic reservoir model, the following results were obtained: $k_x = k_y = 0.041\ \mu\text{m}^2$, $k_z = 0.123 \cdot 10^{-4}\ \mu\text{m}^2$, and $p_{\text{res}} = 8.582$ MPa; for a homogeneous reservoir model, $k = 0.014\ \mu\text{m}^2$. Figure 1b shows the calculated curve of the parameter $s(p)$.

Figure 2a shows the pressure variation (pumping curve) for horizontal well No. 1947 of the Sirenevskoye field. The well production rate before shutdown is $Q = 9.95 \cdot 10^{-5}$ m^3/sec, the length of the horizontal portion of the well is 310 m, the oil viscosity is $\mu = 25$ mPa·sec, the reservoir thickness is 31 m, and the initial approximation of the reservoir pressure is $p^0_{\text{res}} = 3.633$ MPa. Figure 2b shows the calculated curve of the parameter $s(p)$. The calculations showed that $k_x = k_y = 0.031$ μm^2, $k_z = 0.033$ μm^2, $p_{\text{res}} = 3.915$ MPa; for the homogeneous reservoir model, $k = 0.039$ μm^2.

The proposed computational algorithms for interpreting the results of hydrodynamic studies of horizontal wells provide estimates of the reservoir anisotropy, reservoir pressure, and pressure dependences of the filtration characteristics of reservoirs.

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